

BAA008 Matematika I (G)

Derivace

11.3.1 Určete derivaci $f'(x)$ a definiční obory $\mathcal{D}(f), \mathcal{D}(f')$ funkcí:

a) $f(x) = \frac{4x^7 + 3x^5 - 2x^4 + 7x - 2}{3x^4}$

$$[\mathcal{D}(f) = \mathbb{R} - \{0\}, f'(x) = \frac{12x^7 + 3x^5 - 21x + 8}{3x^5}, \mathcal{D}(f') = \mathcal{D}(f)]$$

b) $f(x) = (x^3 + 8)(x - 2)$

$$[\mathcal{D}(f) = \mathbb{R}, f'(x) = 4x^3 - 6x^2 + 8, \mathcal{D}(f') = \mathcal{D}(f)]$$

c) $f(x) = \frac{e^x - 1}{e^x + 1}$

$$[\mathcal{D}(f) = \mathbb{R}, f'(x) = \frac{2e^x}{(e^x + 1)^2}, \mathcal{D}(f') = \mathcal{D}(f)]$$

d) $f(x) = \frac{1}{\log(3x^2 + x + 1)}$

$$[\mathcal{D}(f) = \mathbb{R} - \{0, -\frac{1}{3}\}, f'(x) = -\frac{6x + 1}{(3x^2 + x + 1) \ln 10 \log^2(3x^2 + x + 1)}, \mathcal{D}(f') = \mathcal{D}(f)]$$

11.3.2 Určete první a druhou derivaci $f'(x), f''(x)$ a příslušné definiční obory funkcí:

a) $f(x) = x\sqrt{x^2 + 3}$

$$[f'(x) = \frac{2x^2 + 3}{\sqrt{x^2 + 3}}, f''(x) = \frac{x(2x^2 + 9)}{\sqrt{(x^2 + 3)^3}}, \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R}]$$

b) $f(x) = \ln \sqrt{\frac{1 - \sin x}{1 + \sin x}}$

$$[f'(x) = -\frac{1}{\cos x}, f''(x) = -\frac{\sin x}{\cos^2 x}, \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R} - \{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}]$$

11.3.3 Určete druhou derivaci $f''(x)$ a příslušné definiční obory funkcií:

a) $f(x) = x(\ln x - 1)$

$$[f''(x) = \frac{1}{x}, \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = (0, \infty)]$$

b) $f(x) = \operatorname{arctg}(x - \sqrt{x^2 + 1})$

$$[f''(x) = -\frac{x}{(x^2 + 1)^2}, \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R}]$$

11.3.4 Najděte rovnici tečny t a normály n ke grafu funkce $y = f(x)$:

a) $f(x) = e^{-x} \cos 2x$ v bodě $A = [0, ?]$ [$t : x + y - 1 = 0, n : x - y + 1 = 0$]

b) $f(x) = e^{\frac{x}{2}} + 1$, je-li t rovnoběžná s přímkou $x - 2y + 1 = 0$

[$t : x - 2y + 4 = 0, n : 2x + y - 2 = 0$]