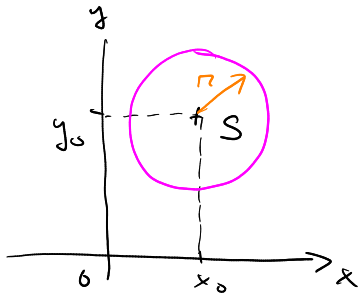
BAA008 Matematika I (G)

Cvičení č. 1

Kuželosečky



A KRUŽNICE $(x-x_0)^2+(y-y_0)^2=r^2$

$S=[x_0,y_0]$ ... STŘED, r ... POLOMĚR

B ELIPSA $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$

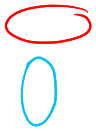
$S=[x_0,y_0]$ ... STŘED

a,b ... POLOOSY

e ... EXCENTRICITA $[a^2=b^2+e^2]$

$a>b$

$a<b$



C HYPERBOLA $\pm\frac{(x-x_0)^2}{a^2}\mp\frac{(y-y_0)^2}{b^2}=1$

$S=[x_0,y_0]$ ... STŘED, a,b ... POLOOSY, e ... EXCENTRICITA $[e^2=a^2+b^2]$

D PARABOLA $(y-y_0)^2=\pm 2p(x-x_0)$

$(x-x_0)^2=\pm 2p(y-y_0)$

$V=[x_0,y_0]$ ... VRCHOL

p ... PARAMETR - VZDÁLENOST OHNISKA OD ŘÍDÍCÍ PŘÍMKY

$$Ax^2+Bx+Cy^2+Dy+E=0$$

$A=C$ $\Rightarrow$ KRUŽNICE

$A\neq C$, A,C - STEJNÁ ZNAMÉNKA $\Rightarrow$ ELIPSA

A,C - OPAČNÁ ZNAMÉNKA $\Rightarrow$ HYPERBOLA

$A=0$ NEBO $C=0$ $\Rightarrow$ PARABOLA

Příklad 0.2.B $B = \{[x, y] \in E_2; x^2 + y^2 + 2x - 6y + 1 > 0,\ \ 64x^2 + 25y^2 - 1600 < 0\}$

$x^2 + y^2 + 2x - 6y + 1 > 0$ $\Rightarrow$ KRUŽNICE $S = [-1, 3]$

$x^2 + 2x + y^2 - 6y + 1 > 0$ $r = 3$

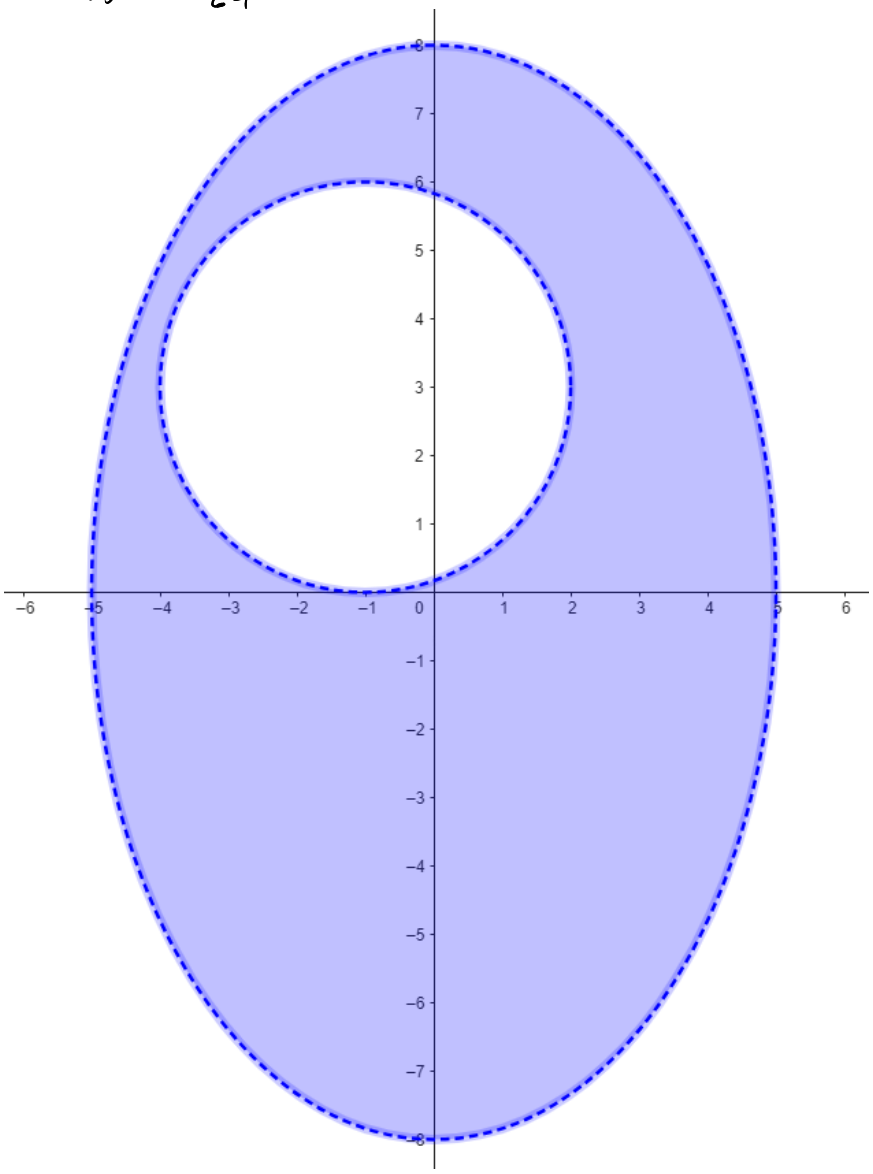
$(x+1)^2 - 1 + (y-3)^2 - 9 + 1 > 0$

$(x+1)^2 + (y-3)^2 > 3^2$

$64x^2 + 25y^2 - 1600 < 0$ $\Rightarrow$ ELIPSA $S = [0, 0]$,

$64x^2 + 25y^2 < 1600 \quad / : 160$ $a = 5,\ b = 8$

$\frac{x^2}{25} + \frac{y^2}{64} < 1$



Příklad 0.6.F $F = \{[x, y] \in E_2; 36x - 9y^2 - 12y - 58 = 0\}$

$$36x - 9y^2 - 12y - 58 = 0$$

$$9y^2 - 12y = 36x - 58 \qquad \Rightarrow \text{PARABOLA} \quad V = \left[\tfrac{3}{2}, -\tfrac{2}{3}\right]$$

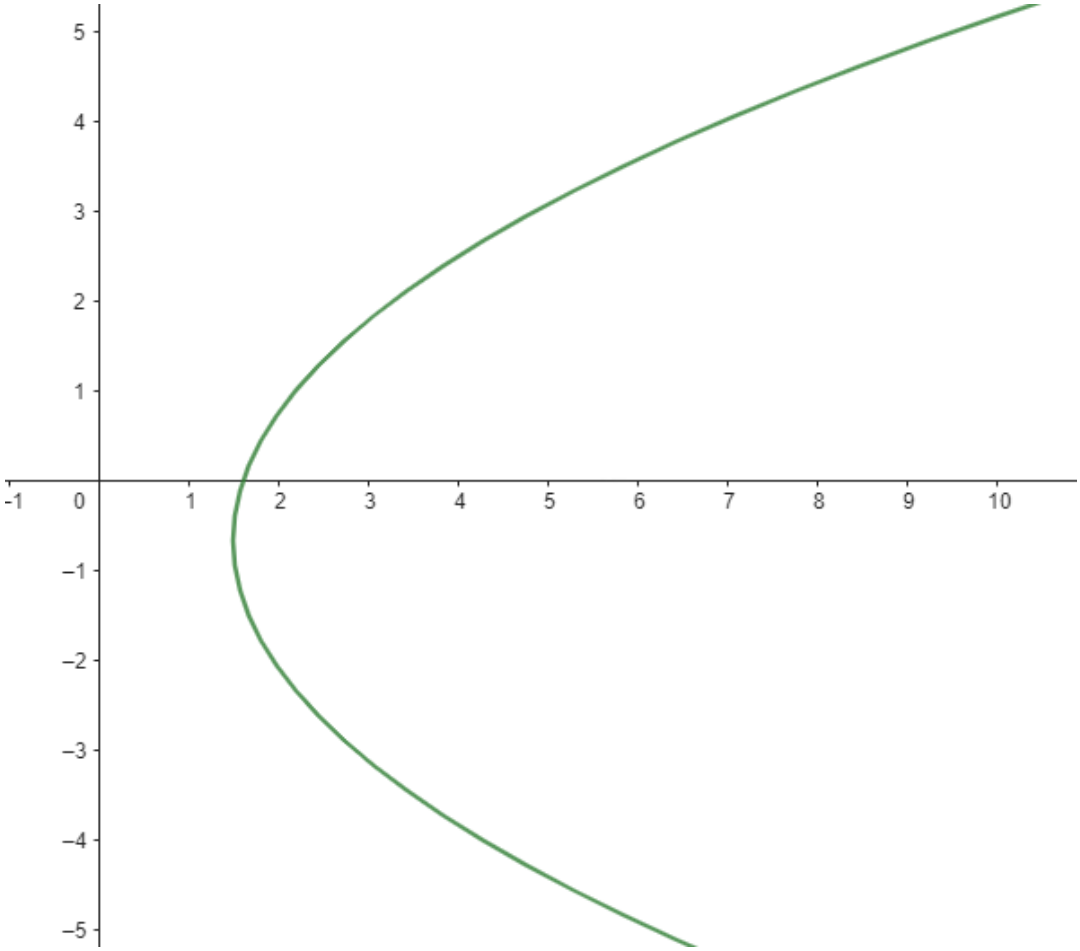
$$9\left(y^2 + \tfrac{4}{3}y\right) = 36x - 58 \qquad p = 2$$

$$9\left(y + \tfrac{2}{3}\right)^2 - 4 = 36x - 58$$

$$9\left(y + \tfrac{2}{3}\right)^2 = 36x - 54 \quad / \tfrac{1}{9}$$

$$\left(y + \tfrac{2}{3}\right)^2 = 4x - 6$$

$$\left(y + \tfrac{2}{3}\right)^2 = 4\left(x - \tfrac{3}{2}\right)$$



Příklad 0.8.K $K = \{[x, y] \in E_2; 9x^2 - 4y^2 - 18x - 16y - 43 = 0\}$

$$9x^2 - 4y^2 - 18x - 16y - 43 = 0$$

$$9x^2 - 18x - 4y^2 - 16y - 43 = 0$$

$$9(x^2 - 2x) - 4(y^2 + 4y) - 43 = 0$$

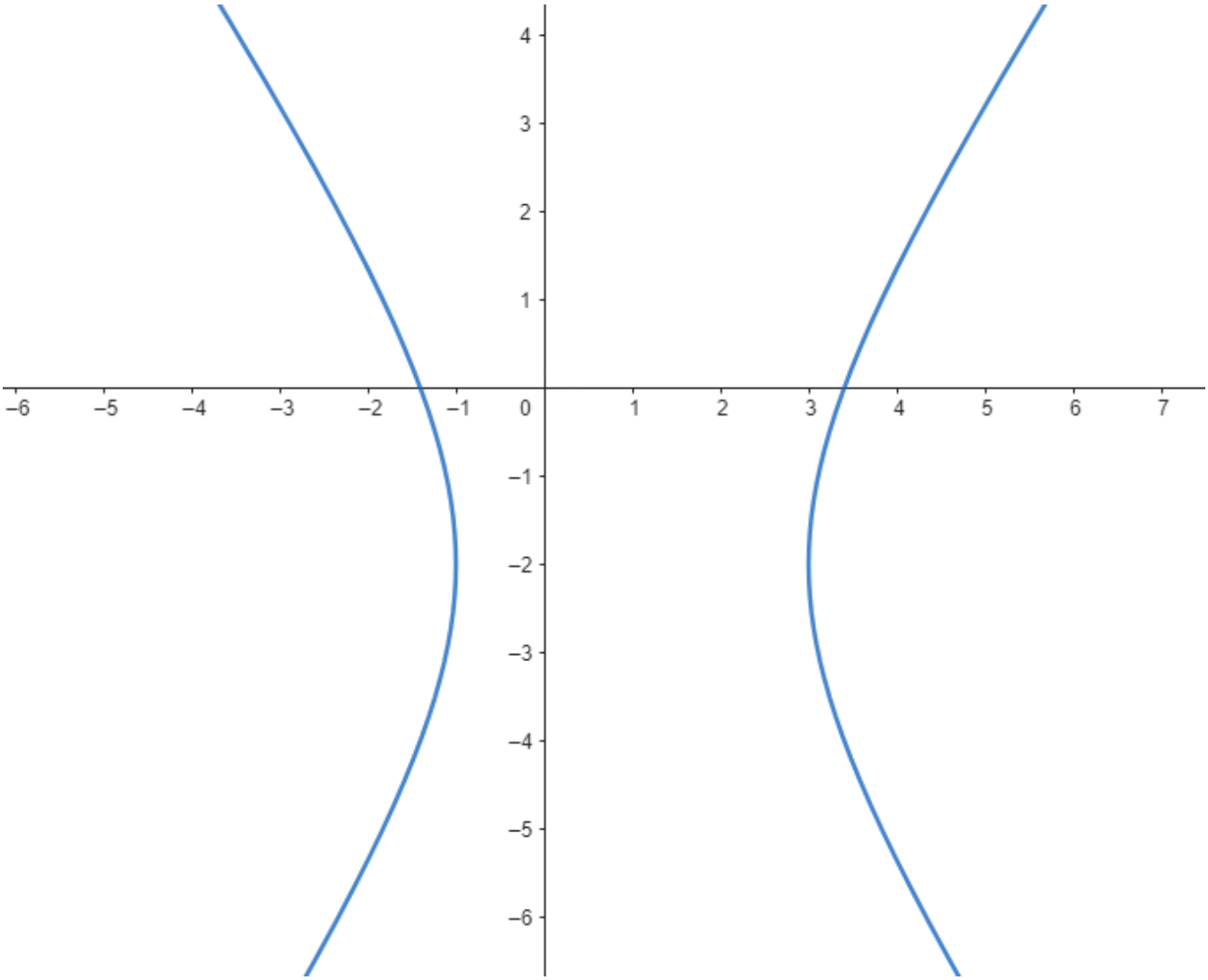
$$9(x-1)^2 - 9 - 4(y+2)^2 + 16 - 43 = 0$$

$$9(x-1)^2 - 4(y+2)^2 = 36 \quad / \cdot \frac{1}{36}$$

$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$$

$\Rightarrow$ HYPERBOLA $S = [1, -2]$

$a = 2,\ b = 3$



Příklad 0.7.G $G = \{[x, y] \in E_2; -4y^2 + 12y - 12x - 15 = 0\}$

$$-4y^2 - 12y - 12x - 15 = 0 \quad \Rightarrow \text{PARABOLA} \quad V = [-\tfrac{1}{2}, \tfrac{3}{2}]$$

$$4y^2 - 12y = -12x - 15 \qquad p = \tfrac{3}{2}$$

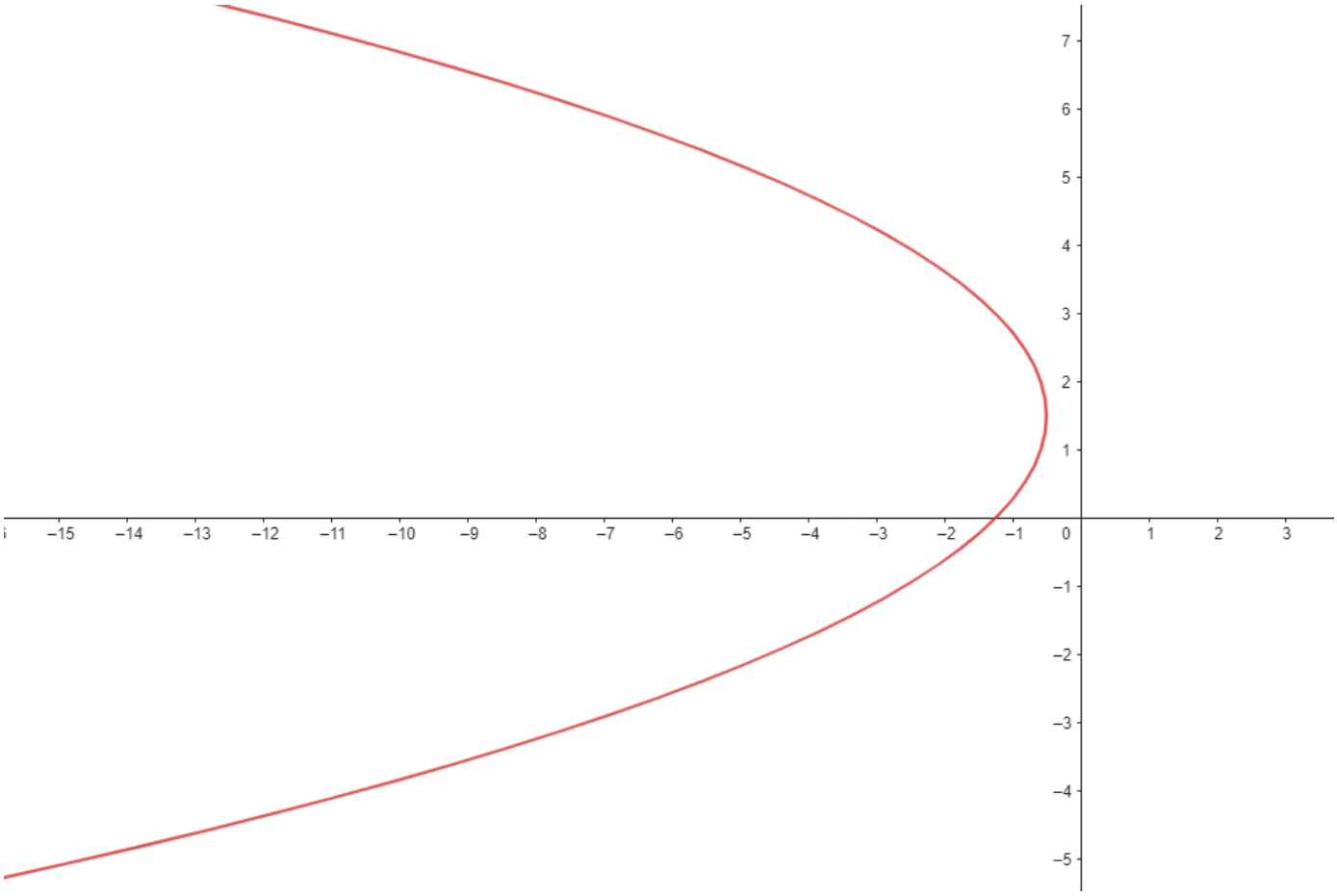
$$4(y^2 - 3y) = -12x - 15$$

$$4(y - \tfrac{3}{2})^2 - 9 = -12x - 15$$

$$4(y - \tfrac{3}{2})^2 = -12x - 6$$

$$(y - \tfrac{3}{2})^2 = -3x - \tfrac{3}{2}$$

$$(y - \tfrac{3}{2})^2 = -3(x + \tfrac{1}{2})$$

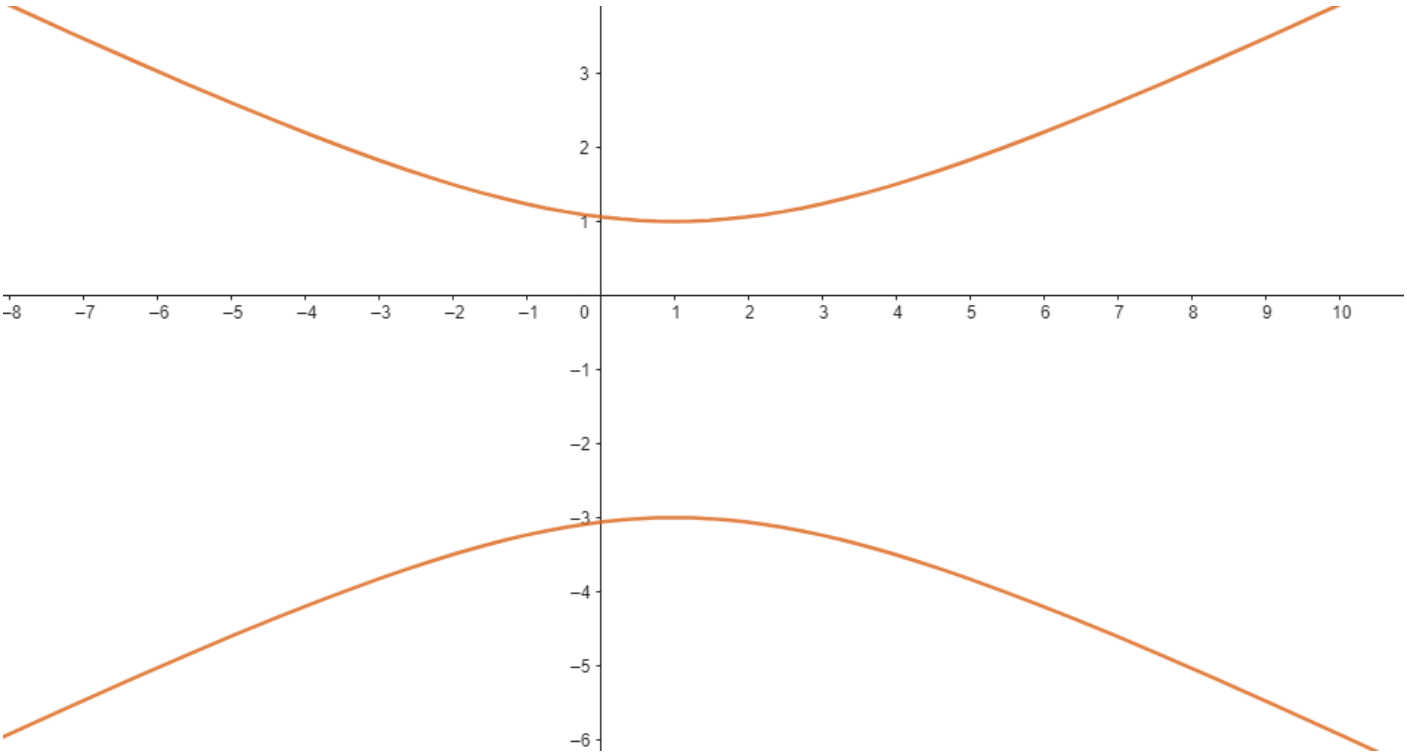


Příklad 0.10.L $L = \{[x, y] \in E_2; 16y^2 - 4x^2 + 32y + 8x - 52 = 0\}$

$$16y^2 - 4x^2 + 32y + 8x - 52 = 0$$
$$-4x^2 + 8x + 16y^2 + 32y - 52 = 0$$
$$-4(x^2 - 2x) + 16(y^2 + 2y) - 52 = 0$$
$$-4(x-1)^2 + 4 + 16(y+1)^2 - 16 - 52 = 0$$
$$-4(x-1)^2 + 16(y+1)^2 = 64 \quad / \cdot \frac{1}{64}$$
$$-\frac{(x-1)^2}{16} + \frac{(y+1)^2}{4} = 1$$

$\Rightarrow$ HYPERBOLA $S = [1, -1]$

$a = 4,\ b = 2$



Absolutní hodnota

- NEZÁPORNÉ REÁLNÉ ČÍSLO $|a|$, PRO KTERÉ PLATÍ:
  1) JE-LI $a \geq 0 \rightarrow |a| = a$
  2) JE-LI $a < 0 \rightarrow |a| = -a$

PLATÍ: $|a \cdot b| = |a| \cdot |b|$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a+b| \leq |a| + |b|$$

$x \in \langle -d, d \rangle \iff |x| \leq d$

$x \in (-\infty, -d) \cup (d, \infty) \iff |x| > d$

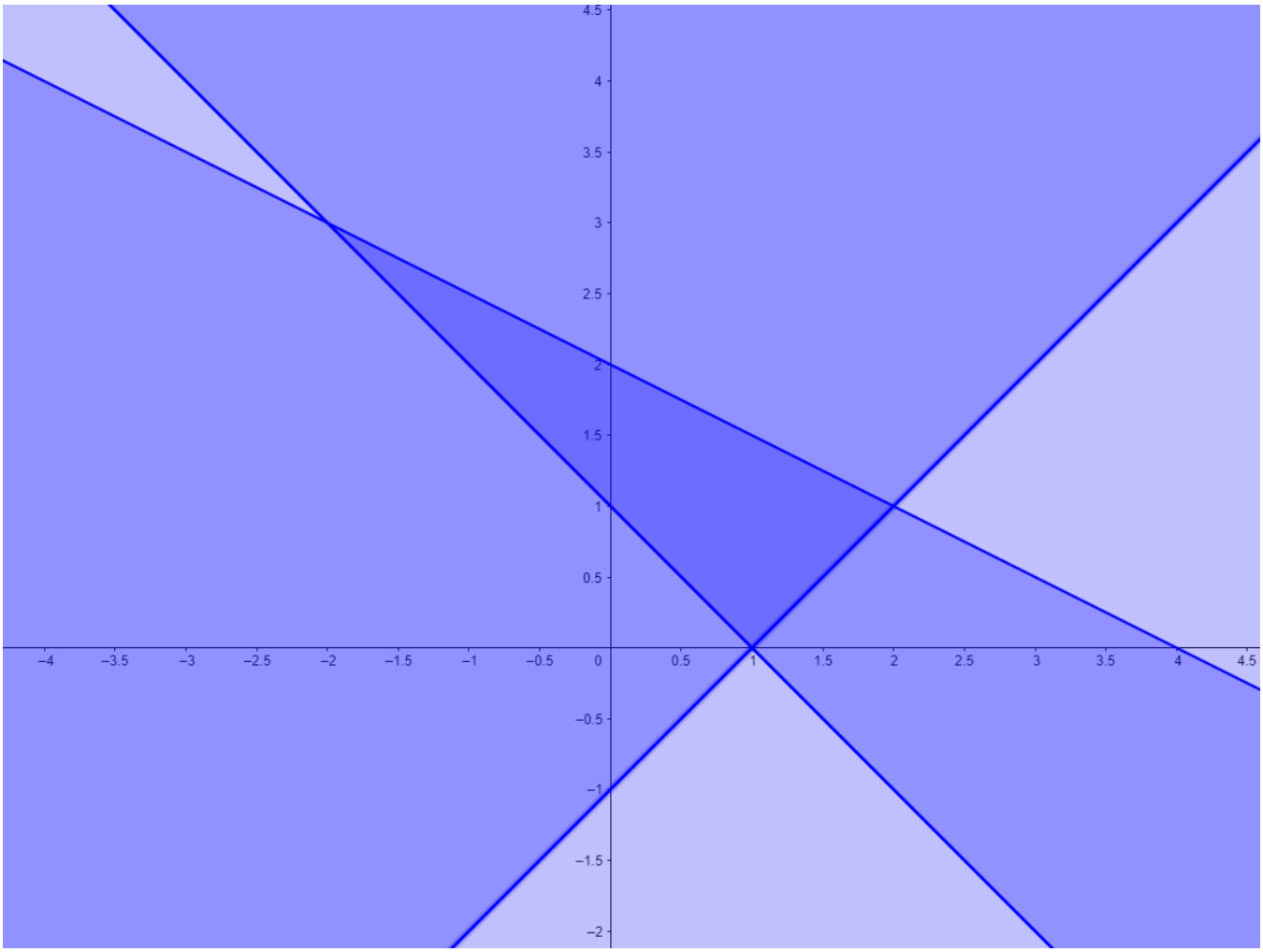
$x \in (c-d, c+d) \iff |x-c| < d$ „VZDÁLENOST ČÍSLA x OD c JE MENŠÍ NEŽ d".

Příklad 0.3.C $C = \left\{ [x, y] \in E_2; |1 - x| \leq y \leq -\frac{1}{2}x + 2 \right\}$

$$|1-x| \leq y \quad \wedge \quad y \leq -\frac{1}{2}x + 2$$

$$-y \leq 1 - x \leq y$$

$$y \geq x - 1 \quad \wedge \quad y \geq 1 - x \quad \wedge \quad y \leq -\frac{1}{2}x + 2$$



Příklad 0.11.M Sestrojte graf funkce $f : y = 2|x-5| + |3x+9| - |1-x| - 15$.

NULOVÉ BODY : $-3, 1, 5$

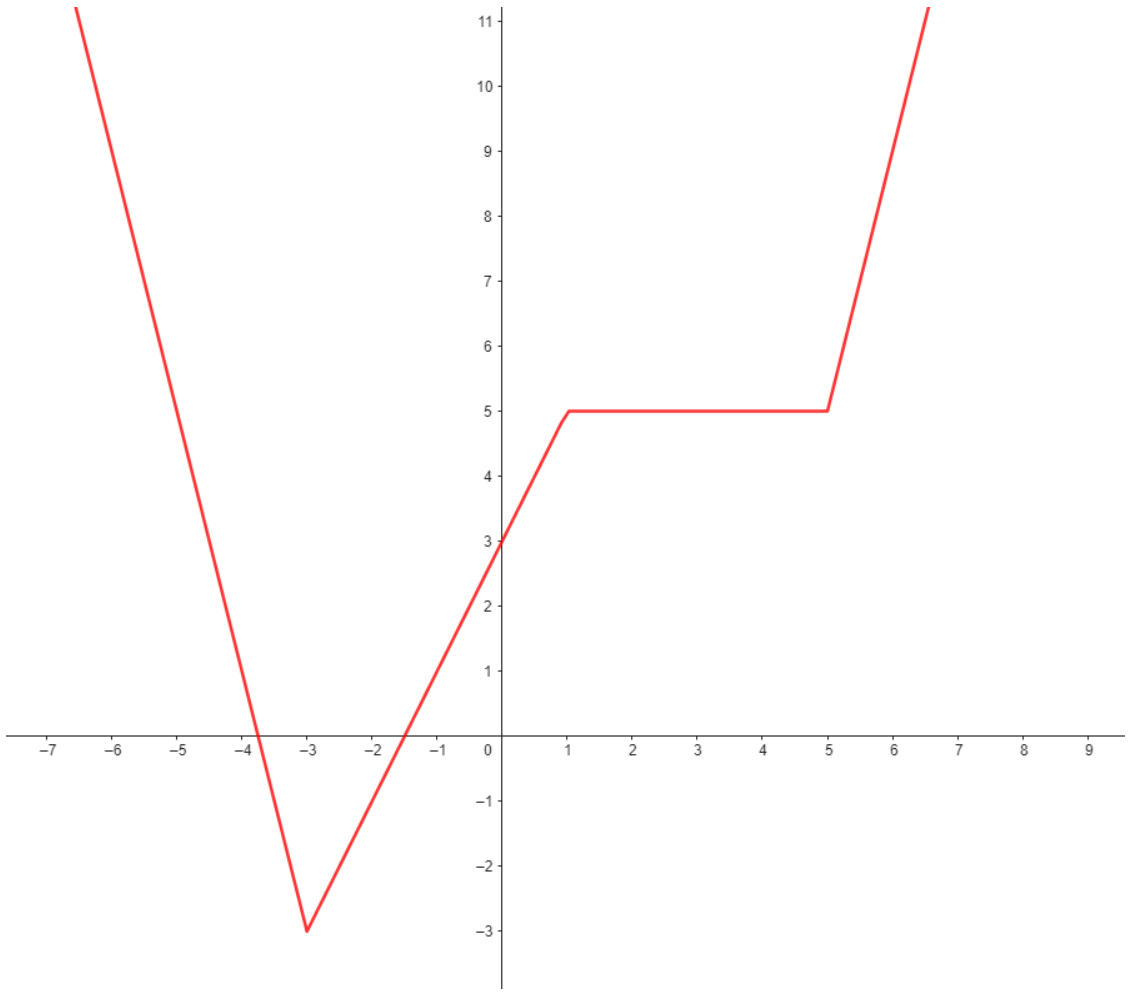
	$(-\infty,-3\rangle$	-3	$(-3,1\rangle$	1	$(1,5\rangle$	5	$(5,\infty$
$x-5$	$-$		$-$		$-$		$+$
$3x+9$	$-$		$+$		$+$		$+$
$1-x$	$+$		$+$		$-$		$-$

$x \in (-\infty,-3\rangle$: $-2(x-5)-(3x+9)-(1-x) = -4x-15$

$x \in (-3,1\rangle$: $-2(x-5)+(3x+9)-(1-x) = 2x+3$

$x \in (1,5\rangle$: $-2(x-5)+(3x+9)-[-(1-x)] = 5$

$x \in (5,\infty\rangle$: $2(x-5)+(3x+9)+(1-x) = 4x-15$



Příklad 0.12.N Řešte nerovnici s absolutní hodnotou $3|x-2|-6|x|<2x-1$.

NULOVÉ BODY: 0, 2

	$(-\infty, 0\rangle$	0	$(0, 2\rangle$	2	$(2, \infty)$
$x-2$	$-$		$-$		$+$
x	$-$		$+$		$+$
	I.		II.		III.

I: $3(2-x)-6(-x) < 2x-1$

$-3x+6x-2x < -1-6$

$x < -7$ $\qquad x \in (-\infty, -7)$

II. $3(2-x)-6x < 2x-1$

$6+1 < 2x+6x+3x$

$\frac{7}{11} < x$ $\qquad x \in (\frac{7}{11}, 2\rangle$

III. $3(x-2)-6x < 2x-1$

$-6-1 < 2x+6x-3x$

$-1 < x$ $\qquad x \in (2, \infty)$

$$\underline{\underline{x \in (-\infty, 7) \cup (\tfrac{7}{11}, \infty)}}$$