# BAA009 Matematika 2 (G)

## Cvičení č. 6

1. zápočtová písemka

**Příklad 6.1.** Určete definiční obor funkce a graficky ho znárorněte v $\mathbb{E}_2$.

a) $f(x,y) = \ln \dfrac{x}{y}$,

$[\ D(f) = \{[x,y] \in \mathbb{E}_2; (x > 0 \wedge y > 0) \vee (x < 0 \wedge y < 0)\}\ ]$

FUNKCE DVOU PROMĚNNÝCH $f: \mathbb{E}_2 \to \mathbb{R}$

$[x,y] \to z \in \mathbb{R}$

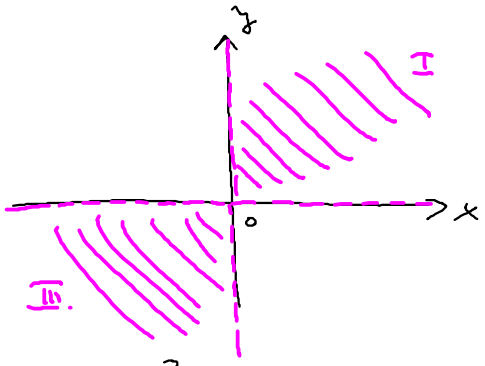
$z = f(x,y)$ 1) $D(f)$ ... DEF. OBOR

2) $\forall\ X = [x,y] \in D(f)$ JE PŘIŘAZENO PRÁVĚ JEDNO REÁLNÉ ČÍSLO $z = f(x,y)$

$f(x,y) = \ln \frac{x}{y}$

$D(f)$: $\frac{x}{y} > 0$ = 1. $x > 0 \wedge y > 0$

2. $x < 0 \wedge y < 0$




$D(f)$: $\{[x,y] \in \mathbb{E}_2;\ (x > 0 \wedge y > 0) \vee (x < 0 \wedge y < 0)\}$

b) $f(x,y) = \sqrt{6x + 4y - x^2 - y^2 - 12}$,

[ $D(f) = \{[x,y] \in \mathbb{E}_2; (x-3)^2 + (y-2)^2 \leq 1$ ]

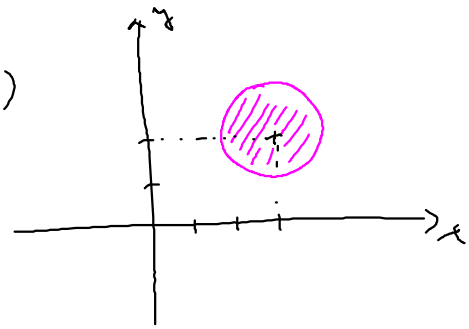
$$f(x,y) = \sqrt{6x + 4y - x^2 - y^2 - 12}$$

$$D(f): \quad 6x + 4y - x^2 - y^2 - 12 \geq 0 \quad / \cdot (-1)$$

$$x^2 - 6x + y^2 - 4y + 12 \leq 0$$

$$(x-3)^2 - 9 + (y-2)^2 - 4 + 12 \leq 0$$

$$(x-3)^2 + (y-2)^2 \leq 1$$




= KRUŽNICE : $k(S = [3,2], r = 1)$

$$D(f) = \{[x,y] \in \mathbb{E}_2; (x-3)^2 + (y-2)^2 \leq 1\}$$

c) $f(x,y) = \dfrac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}$.

[ $D(f) = \{[x,y] \in \mathbb{E}_2; (y^2 \leq 4x) \wedge (x^2 + y^2 < 1) \wedge ([x,y] \neq [0,0])$ ]

$$f(x,y) = \frac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}$$

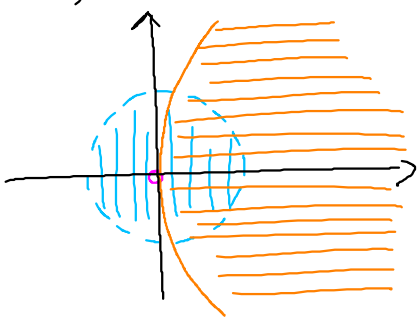
1. $4x - y^2 \geq 0$
$y^2 \leq 4x$
$\frac{1}{4} y^2 \leq x$

2. $1 - x^2 - y^2 > 0$
$x^2 + y^2 < 1$

3. $\ln(1 - x^2 - y^2) \neq 0$
$1 - x^2 - y^2 \neq 1$
$x^2 + y^2 \neq 0$
$x \neq 0 \wedge y \neq 0$
$[0,0] \notin D(f)$

$D(f) = 1. \cap 2. \cap 3.$

$$D(f) = \{[x,y] \in \mathbb{E}_2; (\tfrac{1}{4} y^2 \leq x \wedge x^2 + y^2 < 0 \wedge [0,0] \notin D(f)\}$$



**Příklad 6.2.** Určete parciální derivace prvního řádu funkce.

a) $z = x^2 + 2xy + 3y^2$,

[ $z'_x = 2x + 2y$, $z'_y = 2x + 6y$ ]

PARCIÁLNÍ DERIVACE $A[x_0, y_0]$

$f(x,y)$

$$f'_x(x_0,y_0) = \lim_{h \to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h}$$

$$f'_y(x_0,y_0) = \lim_{k \to 0} \frac{f(x_0, y_0+k) - f(x_0,y_0)}{k}$$

LIMITY MUSÍ EXISTOVAT A MUSÍ BÝT KONEČNÉ!

$f''_{xx}$, $f''_{yy}$, $f''_{xy} = f''_{yx}$

↳ NEZÁLEŽÍ NA POŘADÍ DERIVOVÁNÍ, ZÁLEŽÍ POUZE NA TOM, KOLIKRÁT DERIVUJEME PODLE x A KOLIKRÁT DERIVUJEME PODLE y

$f'''_{xxy} = f'''_{xyx} = f'''_{yxx}$

$z = x^2 + 2xy + 3y^2$

POKUD DERIVUJEME PODLE JEDNÉ PROMĚNNÉ, TAK NA DRUHOU PROMĚNNOU POHLÍŽÍME JAKO NA KONSTANTU.

$z'_x = f'_x(x,y) = 2x + 2y \cdot 1 + 0 = 2x + 2y$

$z'_y = f'_y(x,y) = 0 + 2x \cdot 1 + 3 \cdot 2y = 2x + 6y$

b) $z = x^y$,

$[\ z'_x = yx^{y-1},\ z'_y = x^y \ln x\ ]$

$$z = x^y$$

$$z'_x = y x^{y-1} \qquad (x^n)'$$

$$z'_y = x^y \cdot \ln x \qquad (a^x)'$$

c) $z = \dfrac{3xy}{x-y}$,

$[\ z'_x = -\dfrac{3y^2}{(x-y)^2},\ z'_y = \dfrac{3x^2}{(x-y)^2}\ ]$

$$z = \frac{3xy}{x-y}$$

$$z'_x = \frac{3y(x-y) - 3xy \cdot 1}{(x-y)^2} = \frac{\cancel{3xy} - 3y^2 - \cancel{3xy}}{(x-y)^2} = -\frac{3y^2}{(x-y)^2}$$

$$z'_y = \frac{3x(x-y) - 3xy \cdot (-1)}{(x-y)^2} = \frac{3x^2 - \cancel{3xy} + \cancel{3xy}}{(x-y)^2} = \frac{3x^2}{(x-y)^2}$$

d) $z = ye^{\sqrt{x^2+y^2}}$.

$[\ z'_x = \dfrac{xye^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}},\ z'_y = e^{\sqrt{x^2+y^2}}\left(1 + \dfrac{y^2}{\sqrt{x^2+y^2}}\right)\ ]$

$$z = y e^{\sqrt{x^2+y^2}}$$

$$z'_x = y e^{\sqrt{x^2+y^2}} \cdot \cancel{\tfrac{1}{2}} (x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}x = \frac{xye^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$$

$$z'_y = 1 \cdot e^{\sqrt{x^2+y^2}} + y \cdot e^{\sqrt{x^2+y^2}} \cdot \cancel{\tfrac{1}{2}} (x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}y =$$

$$= e^{\sqrt{x^2+y^2}} + \frac{y^2 e^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} = e^{\sqrt{x^2+y^2}}\left(1 + \frac{y^2}{\sqrt{x^2+y^2}}\right)$$